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TRIGONOMETRÍA

DIRIGIDA

SEMANA 10

PROBLEMA 1

Si se sabe que $x \in \left\langle \frac{19\pi}{60}; \frac{23\pi}{45} \right]$, calcule la suma del máximo y mínimo valor de la expresión:

$$L = 3\tan^2\left(3x - \frac{\pi}{5}\right) + 1$$

A) 11 B) 14 C) 12
D) 10 E) 13

Resolución

Sea $\theta = 3x - \frac{\pi}{5}$; $\frac{19\pi}{60} < x \leq \frac{23\pi}{45}$

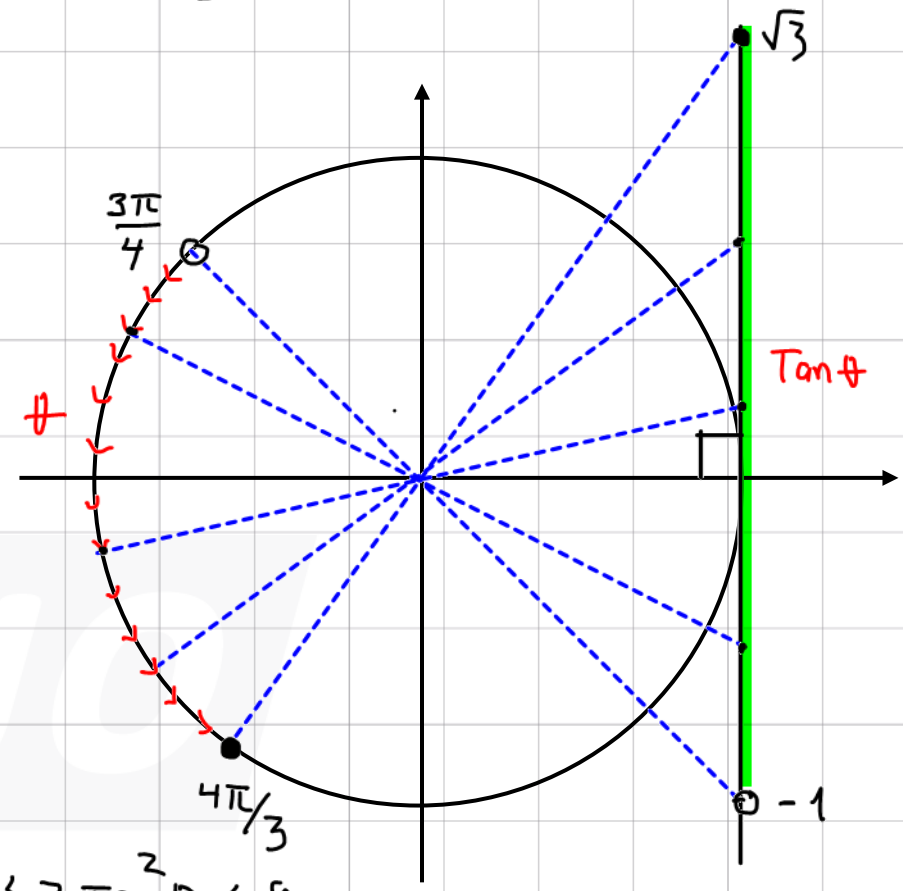
$\frac{19\pi}{20} < 3x \leq \frac{23\pi}{15}$

$\frac{3\pi}{4} < 3x - \frac{\pi}{5} \leq \frac{4\pi}{3}$

$L = 3\tan^2\theta + 1$;

$\frac{3\pi}{4} < \theta \leq \frac{4\pi}{3}$

Del gráfico



$-1 < \tan\theta \leq \sqrt{3}$

$0 \leq \tan^2\theta \leq 3 \rightarrow 0 \leq 3\tan^2\theta \leq 9$

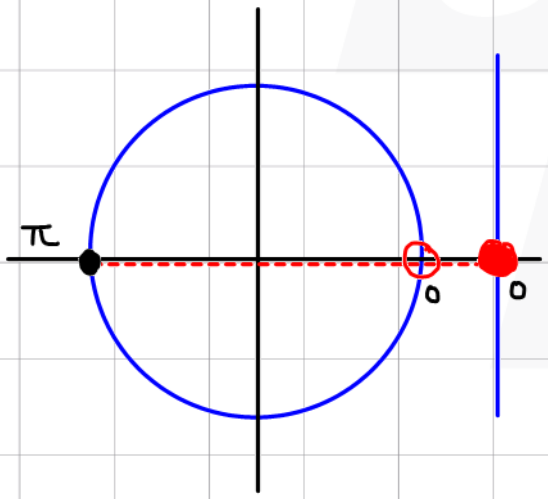
$1 \leq \underbrace{3\tan^2\theta + 1}_{L} \leq 10 \rightarrow L_{máx} = 10 \wedge L_{min} = 1$

$\therefore L_{máx} + L_{min} = 11$

PROBLEMA 2

Si $\operatorname{sen}^2\theta+2\operatorname{sen}\theta=n+2;\ 0<\theta\leq\frac{3\pi}{2}$, calcule el número de valores enteros que adopta n.

A) 3 B) 4 C) 5
D) 6 E) 7



Resolución

Dato

$\operatorname{Sen}^2\theta+2\operatorname{Sen}\theta=n+2;$

$0<\theta\leq\frac{3\pi}{2}$

completamos cuadrados

$\operatorname{Sen}^2\theta+2\operatorname{Sen}\theta+1=n+2+1$

$(\operatorname{Sen}\theta+1)^2-3=n$

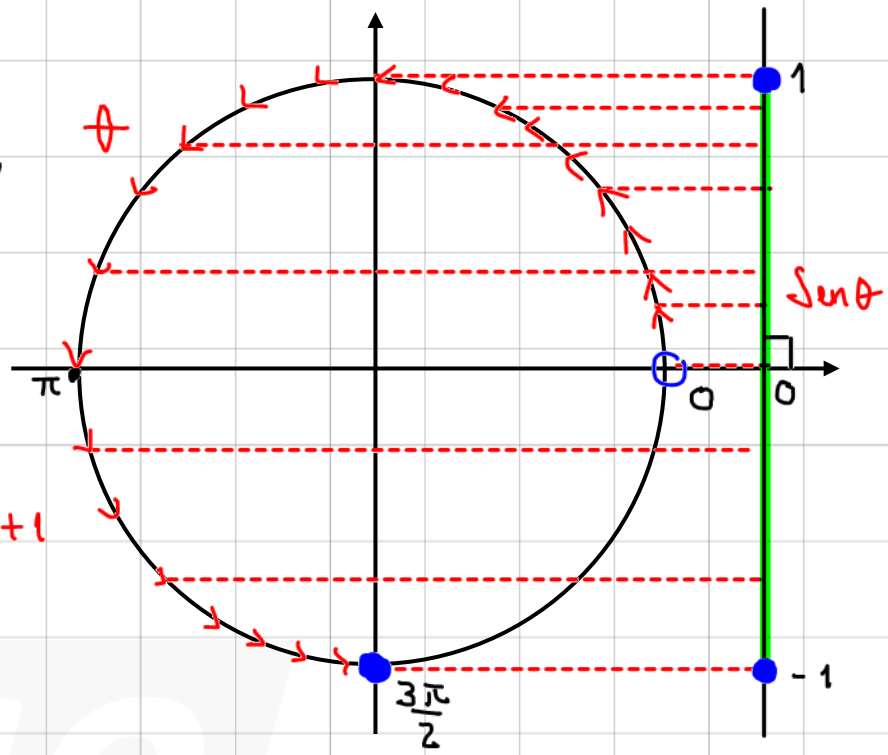
Del gráfico

$-1\leq\operatorname{Sen}\theta\leq 1$

$0\leq\operatorname{Sen}\theta+1\leq 2$

$0\leq(\operatorname{Sen}\theta+1)^2\leq 4$

$-3\leq\underbrace{(\operatorname{Sen}\theta+1)^2-3}_{n}\leq 1$



$n\in\{-3;-2;-1;0;1\}$

Rpta: 5

PROBLEMA 3

Si $0 \le \operatorname{sen}(\alpha) \le \frac{1}{2}; \alpha \in [0;2\pi]$, calcule la variación de $|\tan\alpha|$.

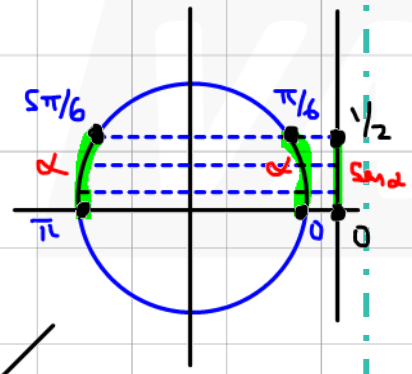
A) $\left[0;\frac{\sqrt{3}}{3}\right]$ B) $\left[\frac{\sqrt{3}}{3};1\right]$ C) $[0;\sqrt{3}]$

D) $[1;\sqrt{3}]$ E) $[0;1]$

$\operatorname{Sen}^2\theta = 1 - \cos^2\theta$

$\cos\theta = \frac{1}{\operatorname{Sec}\theta}$

$\operatorname{Sec}^2\theta = 1 + \operatorname{Tan}^2\theta$



Resolución

Dato

$0 \le \operatorname{sen}\alpha \le \frac{1}{2}$; $0 \le \alpha \le 2\pi$

$0 \le \operatorname{Sen}^2\alpha \le \frac{1}{4}$

$0 \le 1 - \cos^2\alpha \le \frac{1}{4}$

$-1 \le -\cos^2\alpha \le -\frac{3}{4}$

$1 \ge \cos^2\alpha \ge \frac{3}{4}$

$1 \le \frac{1}{\cos^2\alpha} \le \frac{4}{3}$

$1 \le \operatorname{Sec}^2\alpha \le \frac{4}{3}$

$1 \le 1 + \operatorname{Tan}^2\alpha \le \frac{4}{3}$

$0 \le \operatorname{Tan}^2\alpha \le \frac{1}{3}$

$\sqrt{0} \le \sqrt{\operatorname{Tan}^2\alpha} \le \sqrt{\frac{1}{3}}$

$0 \le |\operatorname{Tan}\alpha| \le \frac{\sqrt{3}}{3}$

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PROBLEMA 4

Para que valores de α, se cumple:
$x^2-(2\cot(\alpha))x+1=0$; $x>0$ y $0<\alpha<\pi$

$x>0 \Rightarrow x \in \mathbb{R}$

A) $\left\langle 0;\frac{\pi}{4}\right] \cup \left\langle \frac{\pi}{2};\frac{3\pi}{4}\right]$

B) $\left\langle 0;\frac{\pi}{4}\right]$

C) $\left\langle 0;\frac{\pi}{4}\right\rangle \cup \left\langle \frac{3\pi}{4};\pi\right\rangle$

D) $\left[\frac{\pi}{4};\frac{3\pi}{4}\right]$

E) $\left\langle \frac{\pi}{4};\frac{3\pi}{4}\right\rangle$

Si $x>0$

$\cot\alpha=1 \rightarrow x^2-2x+1=0$

$\rightarrow x=1$

$\cot\alpha=-1 \rightarrow x^2+2x+1=0$

$\rightarrow x=-1$

Resolución

Dato: $x^2-2\cot\alpha \cdot x+1=0$; $x \in \mathbb{R}$

Las raíces de la ecuación cuadrática son reales

$\Rightarrow \Delta \geq 0$

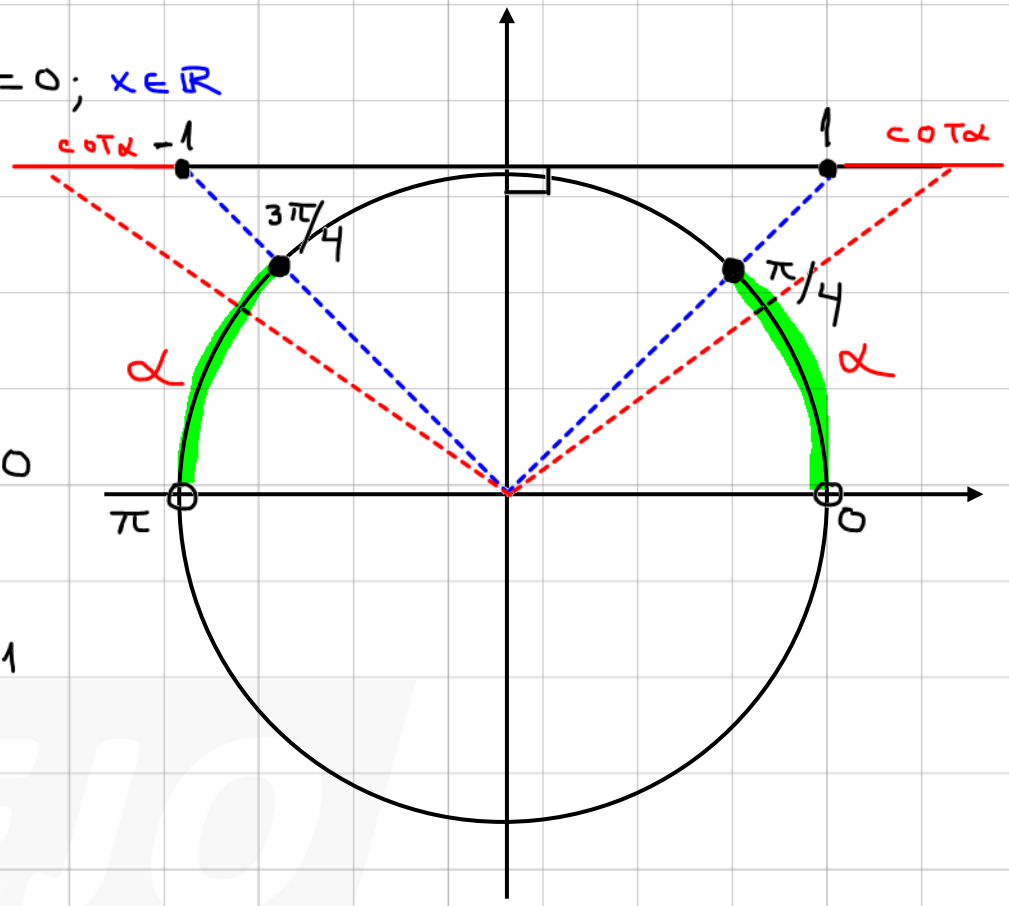
$\Rightarrow (-2\cot\alpha)^2-4(1)(1) \geq 0$

$4\cot^2\alpha \geq 4 \Rightarrow \cot^2\alpha \geq 1$

$\sqrt{\cot^2\alpha} \geq \sqrt{1}$

$\Rightarrow |\cot\alpha| \geq 1$

$\Rightarrow \cot\alpha \geq 1 \vee \cot\alpha \leq -1$



$\Rightarrow \alpha \in \left\langle 0;\frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4};\pi\right\rangle$

PROBLEMA 5

Si $\theta \in IIC$ y $E = \dfrac{\operatorname{sen}\theta + 2}{\operatorname{sen}\theta + 1}$, determine la variación de E^2.

A) $\left\langle \frac{9}{2}; 10 \right\rangle$ B) $\left\langle \frac{\sqrt{3}}{5}; \frac{2}{5} \right\rangle$ C) $\left\langle \frac{3}{4}; \frac{7}{4} \right\rangle$

D) $\left\langle \frac{3}{5}; \frac{7}{5} \right\rangle$ E) $\left\langle \frac{9}{4}; 4 \right\rangle$

Resolución

Dato

$$E = \frac{\operatorname{Sen}\theta + 2}{\operatorname{Sen}\theta + 1} \; ; \; \theta \in IIC$$

$$E = \left(\frac{\operatorname{Sen}\theta + 2}{\operatorname{Sen}\theta + 1} - 1 \right) + 1$$

$$E = \frac{1}{\operatorname{Sen}\theta + 1} + 1$$

$\theta \in IIC \Rightarrow 0 < \operatorname{Sen}\theta < 1$

$1 < \operatorname{Sen}\theta + 1 < 2$

$1 > \dfrac{1}{\operatorname{Sen}\theta + 1} > \dfrac{1}{2}$

$2 > \dfrac{1}{\operatorname{Sen}\theta + 1} + 1 > \dfrac{3}{2}$

$2 > E > \dfrac{3}{2}$

$4 > E^2 > \dfrac{9}{4}$

$\therefore E^2 \in \left\langle \frac{9}{4}; 4 \right\rangle$

PROBLEMA 6

Si $x \in \langle -\sqrt{3}; 1 \rangle$, calcule la variación de

$$N = \frac{2 - \cos^2 x}{1 + \cos^2 x}$$

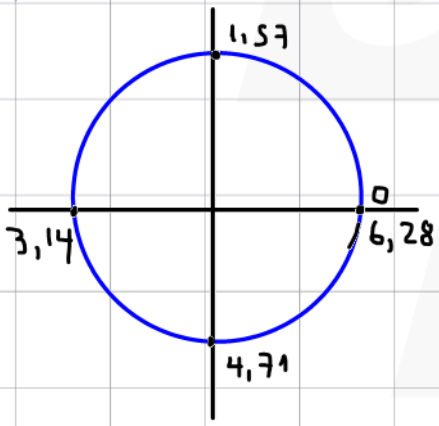
A) $\left[\frac{1}{2}; 2\right]$

B) $\left[\frac{3}{4}; 1\right]$

C) $\left[\frac{1}{2}; \frac{3}{4}\right]$

D) $\left[\frac{1}{3}; \frac{9}{4}\right]$

E) $\left[\frac{1}{3}; \frac{9}{4}\right]$



Resolución

$$N = \left(\frac{2 - \cos^2 x}{1 + \cos^2 x} + 1\right) - 1$$

$$N = \frac{3}{1 + \cos^2 x} - 1 \; ; \; -\sqrt{3} < x < 1$$

Del gráfico

$$\underbrace{\cos(-\sqrt{3})}_{-} \leq \cos x \leq \underbrace{1}_{+}$$

$$0 \leq \cos^2 x \leq 1$$

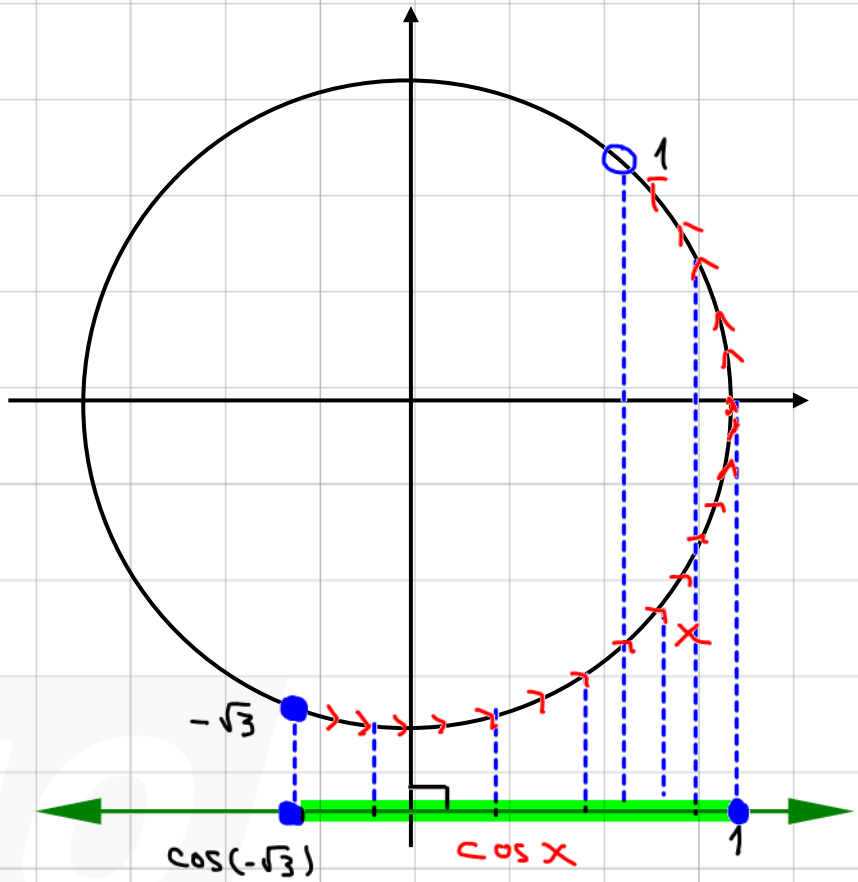
$$1 \leq 1 + \cos^2 x \leq 2$$

$$1 \geq \frac{1}{1 + \cos^2 x} \geq \frac{1}{2} \rightarrow 3 \geq \frac{3}{1 + \cos^2 x} \geq \frac{3}{2}$$

$$2 \geq \frac{3}{1 + \cos^2 x} - 1 \geq \frac{1}{2}$$

N

$$\therefore N \in \left[\frac{1}{2}; 2\right]$$



PROBLEMA 7

Si $\frac{\tan^2\theta-2}{\operatorname{sen}\theta-1}>0;\theta\in\left\langle 0;\frac{\pi}{2}\right\rangle$, calcule la extensión de $\csc^2\theta$.

A) $\left\langle \frac{5}{2};+\infty\right\rangle$ B) $\langle 3;+\infty\rangle$ C) $\langle 2;+\infty\rangle$

D) $\left\langle \frac{4}{3};+\infty\right\rangle$ E) $\left\langle \frac{3}{2};+\infty\right\rangle$

Resolución

$$\frac{\tan^2\theta-2}{\operatorname{sen}\theta-1}>0 \quad ; \quad 0<\theta<\frac{\pi}{2}$$

$\operatorname{sen}\theta-1$ es negativo

$$\Rightarrow \tan^2\theta-2<0 \quad \Rightarrow \quad \underbrace{\tan^2\theta}_{+}<\underbrace{2}_{+}$$

$$\frac{1}{\tan^2\theta}>\frac{1}{2} \quad \Rightarrow \quad \cot^2\theta>\frac{1}{2}$$

$$\cot^2\theta+1>\frac{1}{2}+1 \quad \Rightarrow \quad \csc^2\theta>\frac{3}{2}$$

$$\therefore \csc^2\theta\in\left\langle \frac{3}{2};+\infty\right\rangle$$

Problema 1

Resolución

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